Hysteretic method for measuring the flux trapped within the core of a superconducting lead-coated ferromagnetic torus by a linked superconducting tin ring, in a novel Aharonov-Bohm-like effect based on the Feynman path-integral principle

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Abstract

A novel kind of nonlocal, macroscopic Aharonov-Bohm effect involving two topologically linked superconducting rings made out of two different materials, namely, lead and tin, is suggested for experimental observation, in which the lead ring is a torus containing a core composed of permanently magnetized ferromagnetic material. It is predicted that the remnant fields in a hysteresis loop induced by the application of a magnetic field imposed by a large external pair of Helmholtz coils upon the tin ring, will be asymmetric with respect to the origin of the loop. An appendix based on Feynman's path-integral principle is the basis for these predictions.

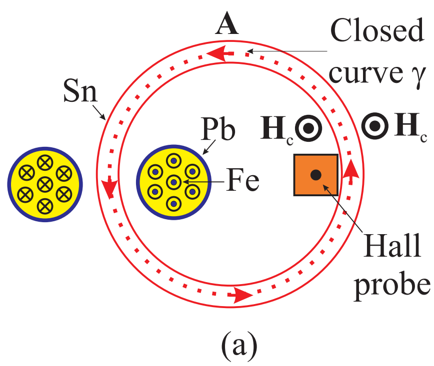

Figure 1: (a) Two topologically-linked superconducting (SC) rings are made out of two different SC materials, tin (Sn) and lead (Pb). The lead ring (i.e., the torus shown in a cross sectional view with a blue rim, and with the black dots and crosses denoting trapped ferromagnetic flux filling its interior) has the higher transition temperature and field, but the tin ring (the circle in red) has the lower transition temperature and field. The lead torus (outlined in blue) encloses within it a magnetized ferromagnetic (Fe) core (filled in with yellow), and traps within its interior a field of around 200 Gauss, which points in the *same* direction as the externally applied critical field $\mathbf{H}_c$ of tin. (b) Same as (a), except that the torus traps within its ferromagnetic core a field pointing in the *opposite* direction to the applied critical field $\mathbf{H}_c$ of tin. The Hall probe measures the remnant field trapped within the tin ring at the zero-crossings of the applied magnetic field $\mathbf{H}$ for both (a) and (b). $\mathbf{A}$ denotes the *total* vector potential evaluated at the dashed circle from *all* sources.

In Bong-Soo Kang's experiment, in addition to the observation of the angular momentum associated with the vector potential, there is yet another interesting possibility for observing and measuring another kind of Aharonov-Bohm-like effect (see Figure 1). If the tin ring in Figure 1 is much thicker than a penetration depth, then the quantization of flux trapped inside this ring will obey the following condition:

$$\Phi_n = \oint_\gamma \mathbf{A} \cdot d\mathbf{l} = n\frac{h}{2e}, n = 0, \pm 1, \pm 2, ... \qquad (1)$$

where Φ_n is the trapped flux, $\mathbf{A}$ is the *total* vector potential arising from *all* sources evaluated along the closed curve γ deep within the material of the tin ring (e.g., the dashed circle γ in Figure 1), $d\mathbf{l}$ is a line element of this closed curve γ, n is an integer, h is Planck's constant, and e is the electron charge.

In the space enclosed by the tin ring in Figure 1, there is a superconducting lead-coated torus containing a ferromagnetic core. This kind of topological configuration, which has a nontrivial topological linking number of unity, is isomorphic to that of the ferromagnetic torus overcoated by

superconducting niobium used by Tonomura et al., by which they convincingly demonstrated the Aharonov-Bohm effect [2]. The superconducting lead coating of the torus will quantize the magnetic flux contained within the ferromagnetic core of the torus. However, more importantly, the magnetic field arising from the ferromagnetic material within the torus will be completely shielded by the Meissner effect due to the thick overcoating of the superconducting lead [3], and will thereby be prevented from leaking into the space exterior to the torus in which the tin ring resides. Hence from a purely classical viewpoint, the electrons within the tin ring can feel no forces arising from the ferromagnetic field which is entirely confined to the interior of the superconducting lead torus. Therefore, classically, there could exist no influence whatsoever of the ferromagnetic field *interior* to the lead torus upon the tin ring *exterior* to the lead torus. If any such influence were to be observed in our experiments, it would imply a violation of the principle of *locality* implicit in classical physics. A positive result in our experiments would constitute evidence for a new kind of macroscopic quantum *nonlocality*, which arises from a Aharonov-Bohm-type quantum holonomy that is entirely nonclassical in origin and nature (see Appendix).

At the quantum level of description, there will arise a contribution of the vector potential along the dashed circle γ in Figure 1 due to the trapped ferromagnetic flux contained within the torus, which γ encloses, to the accumulated quantum mechanical phase of the Cooper pair wavefunction. This phase shift arises from the Aharonov-Bohm effect.

Now consider a hysteresis-loop procedure in which we apply an external magnetic field from a large pair of Helmholtz coils upon the entire assembly shown in Figure 1, starting initially from zero field. Let us assume that the ambient temperature is close to absolute zero, and that there is initially no magnetic field trapped by the tin ring. Then when the applied external magnetic field reaches the critical field of tin [4], i.e.,

$$H_{\mathrm{c}} \approx 280\mathrm{Gauss} \tag{2}$$

the tin ring will be driven into the normal state, and the externally applied magnetic field will begin to fill the space enclosed by the tin ring (except for the area already filled by the superconducting lead-coated ferromagnetic torus), so that there will now be two contributions to the vector potential

$$\mathbf{A} = \mathbf{A}_1 + \mathbf{A}_2 \tag{3}$$

where $\mathbf{A}_1$ is the contribution arising from the flux contained within the ferromagnetic core of the superconducting lead-coated torus, and $\mathbf{A}_2$ is the contribution from the flux that now uniformly fills the rest of the space enclosed by the tin ring due to the externally applied magnetic field, whose magnitude will slightly exceed the critical field H_{c} of the tin material of the ring.

If the externally applied magnetic field from the pair of Helmholtz coils is subsequently reduced down below H_{c}, then the tin ring will once again become superconducting, and therefore the tin ring will trap a certain

number of quantized flux lines given by n_{remnant} where

$$\Phi_n|_{\text{trapped}} = \oint\limits_{\gamma} (\mathbf{A}_1 + \mathbf{A}_2) \cdot d\mathbf{l} = n_{\text{remnant}} \frac{h}{2e} \tag{4}$$

If the externally applied magnetic field from the Helmholtz coils is now further reduced down to zero, the integer n_{remnant} will still remain a constant, and therefore there will still remain a remnant flux trapped within the tin ring corresponding to the constant integer n_{remnant}, which can then be measured by the Hall probe in Figure 1 at the zero-crossing of the applied magnetic field.

Note that the integer n_{remnant} will depend on the relative direction of the ferromagnetic field trapped in the *interior* of the lead-coated torus, relative to the direction of the *exterior*, applied critical field H_c, as illustrated in the two cases (a) and (b) in Figure 1. In other words, the integer n_{remnant} in (4) will depend on whether the relative contributions from $\mathbf{A}_1$ and $\mathbf{A}_2$ will reinforce each other, or whether they will tend to cancel each other out. This should result in an *asymmetric* hysteresis loop with *asymmetric* positive and negative remnant fields, as the externally applied magnetic field from the Helmholtz coils is cycled periodically, and symmetrically, from $+H'$ to $-H'$, where $|H'|$ slightly exceeds H_c.

Appendix: Feynman path-integral principle

The Feynman path-integral principle begins with the fact that for any particle, e.g., the electron, propagating through spacetime

$$d\phi \propto dS \tag{5}$$

where, when the electron is viewed as being a *propagating wave*, $d\phi$ is the infinitesimal *phase* acquired by the electron due to its propagation through spacetime along an infinitesimal spacetime path element dx^μ ($\mu = 0, 1, 2, 3$), and where, when the electron is viewed as being a *moving particle*, dS is the infinitesimal *action* acquired by the same electron due to its motion along the same infinitesimal spacetime path element dx^μ. Thus the relationship $d\phi \propto dS$ is an expression of the *wave-particle duality* of the electron, with $d\phi$ being a *wave* property of the electron, and dS being a *particle* property of the electron.

The proportionality constant in the relationship $d\phi \propto dS$ must be determined by experiment, and turns out to be $1/\hbar$, where $\hbar$ is the reduced Planck's constant, so that (5) becomes

$$d\phi = \frac{1}{\hbar} dS \tag{6}$$

For the special case of a free electron propagating through the vacuum in the absence of any external forces

$$d\phi = k_\mu dx^\mu \tag{7}$$

where k_μ is the four-wavevector of the free electron's plane-wave solution (i.e., the solution of the Dirac equation in a field-free vacuum), and where dx^μ is an infinitesimal four-displacement, i.e., an infinitesimal spacetime path element along which the free electron is traveling. It then follows that

$$d\phi = \frac{1}{\hbar} dS = k_\mu dx^\mu \tag{8}$$

and therefore that

$$dS = \hbar k_\mu dx^\mu = p_\mu dx^\mu \tag{9}$$

where

$$p_\mu = \hbar k_\mu \tag{10}$$

is the four-momentum of the electron. From (10), it follows that

$$\mathbf{p} = \hbar \mathbf{k} \tag{11}$$

$$E = \hbar \omega \tag{12}$$

where $\mathbf{p}$ is the three-momentum of the electron, and $\mathbf{k}$ is its three-wavevector, and where E is the energy of the electron, and ω is its angular frequency. The first of these two relationships, (11), is the De Broglie law for the electron, and the second of these two relationships, (12), is the Planck-Einstein relationship, which was first discovered in connection with the photon, but also applies to the electron.

Note that the above phase-action relationship (6) would apply to any neutral particle, as well as to the charged electron. Is there any extra contribution to the phase or the action arising from the fact that the electron is charged? Experimentally, we know that the answer has to be yes to this question, because we know that the electron interacts with electromagnetic fields via its charge $q = e$. Hence we must add to the action (5) an extra piece that describes the interaction of the charge q of a charged particle with an externally applied electromagnetic field, i.e.,

$$dS_q \propto q \tag{13}$$

since we expect the size of the charge-field interaction to be proportional to the size of the charge q. Next, we seek an action dS_q which satisfies the *linearity* requirement

$$dS_q \propto dx^\mu \tag{14}$$

for infinitesimal four-displacements dx^μ of the charge q through spacetime in the presence of electromagnetic fields. This requirement follows from the fact that all physically reasonable fields must become *uniform* fields when viewed on the tiny, infinitesimal length scales given by dx^μ. Hence the action of displacing a charge in the presence of such uniform fields by an infinitesimal amount dx^μ must be *linear* in dx^μ. Putting (13) and (14) together, we therefore get

$$dS_q \propto q dx^\mu \tag{15}$$

However, note that, whereas dS_q is a four-scalar and the charge q is also a four-scalar, dx^μ is, by contrast, a four-vector. Following the spacetime symmetry argument given by Landau and Lifshitz [6], one must therefore

contract the contravariant four-vector dx^μ with some covariant four-vector A_μ in order to be able to form the invariant four-scalar

$$dS_q \propto qA_\mu dx^\mu \tag{16}$$

The only covariant four-vector that can satisfy Landau and Lifshitz's symmetry requirement, and also yield the correct non-relativistic limit [5], is the *electromagnetic* four-vector potential A_μ.

By taking the non-relativistic limit, and by obtaining the non-relativistic Lorentz force law from this limit [1], one also uniquely determines the sign and the proportionality constant of (16). One then finds that in SI units [1]

$$dS_q = +qA_\mu dx^\mu \tag{17}$$

This argument also uniquely determines the minimal coupling rule for quantum mechanics [1].

Let us now integrate (17) along some specified spacetime path from event a to event b

$$S_q\left[\text{path}\right]\Big|_a^b = \int qA_\mu dx^\mu \left[\text{path}\right]\Bigg|_a^b \tag{18}$$

The phase accumulated by the charge q along this path from a to b will be given by

$$\phi_q\left[\text{path}\right]\Big|_a^b = \frac{1}{\hbar}\,S_q\left[\text{path}\right]\Big|_a^b = \frac{q}{\hbar}\int A_\mu dx^\mu \left[\text{path}\right]\Bigg|_a^b \tag{19}$$

At this point, it may be objected that it is always possible to choose a gauge such that the accumulated phase $\phi_q\left[\text{path}\right]\big|_a^b$ becomes identically zero, because one could in principle always arbitrarily choose a scalar function χ such that

$$A_\mu \to A'_\mu = A_\mu + \partial_\mu \chi = 0 \tag{20}$$

at each point along the specified path between a and b in (19).

However, when there is a *closed* path γ which *encloses* a nonzero amount of magnetic flux, such as in the configuration illustrated in Figure 1, there arises a *quantum holonomy* which prevents this from happening, viz.,

$$\phi_q\left[\gamma\right] = \frac{1}{\hbar}S_q\left[\gamma\right] = \frac{q}{\hbar}\oint_\gamma A_\mu dx^\mu \neq 0 \tag{21}$$

From Stokes's theorem, one finds that

$$\phi_q\left[\gamma\right] = \frac{q}{\hbar}\oint_\gamma A_\mu dx^\mu = \frac{q}{\hbar}\Phi_\gamma \neq 0 \tag{22}$$

where the magnetic flux Φ_γ enclosed by the closed curve γ is given by

$$\Phi_\gamma = \oint_\gamma A_\mu dx^\mu \neq 0 \tag{23}$$

which is a gauge-invariant quantity.

If one further chooses the closed curve γ to lie deep inside the superconducting material of the tin ring, such as the dashed red circle of Figure 1, and if one sets $q = 2e$ to be the charge of the Cooper pairs deep inside the superconductor, then it follows from the *single-valuedness* of the Ginzburg-Landau superconducting order parameter that the magnetic flux Φ_γ enclosed by γ must be quantized in integer units of the fundamental flux quantum, i.e.,

$$\Phi_\gamma = \oint_\gamma \mathbf{A} \cdot d\mathbf{l} = n \frac{h}{2e} \tag{24}$$

where n is an integer, and therefore one recovers the flux quantization condition (1). The rest of the above argument in the main body of the paper for a novel, macroscopic manifestation of the Aharonov-Bohm effect then follows.

References

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[3] We shall assume that the externally applied magnetic field from the pair of Helmholtz coils during the hysteresis-loop procedure will always remain less than the critical field of lead near absolute zero temperature, viz., 780 Gauss. See D.L. Decker, D.E. Mapother, and R.W. Shaw, "Critical field measurements of superconducting lead isotopes," Phys. Rev. **112**, 1888 (1958).

[4] R.W. Shaw, D.E. Mapother, and D.C. Hopkins, "Critical fields of tin, indium, and tantalum," Phys. Rev. **120**, 88 (1960).

[5] Note that by taking the non-relativistic limit and deriving the Lorentz force law from (16) (see [1]), one sees that the four-vector potential A_μ in (16) must reduce to the usual electromagnetic three-vector potential $\mathbf{A}$ in the non-relativistic limit. This uniquely identifies the four-vector potential A_μ in (16) as being the *electromagnetic* vector potential.

[6] L.D. Landau and E.M. Lifshitz, *The Classical Theory of Fields*, 4th edition: Volume 2 (Course of Theoretical Physics Series; Butterworth-Heinemann, Oxford, 2000).